aid of the Legendre-Clebsch necessary condition. This condition may be written (for a minimum)

$$\mu_v \cos\alpha + \mu_\gamma(\sin\alpha/v) \geq 0 \tag{2}$$

In solving the variational boundary value problem, it must be ascertained that inequality (2) is satisfied at each point of the solution.

An intermediate control (i.e., not satisfying the desired boundary conditions) is shown in Fig. 1. As the iterations proceed, the control is reshaped until at some point (A in Fig. 1), α† assumes the value of 90°. At this point $\mu_v = 0$ and $\mu_\gamma > 0$ so that the Legendre-Clebsch condition reduces to

$$C \sin\alpha \geq 0 \qquad C > 0 \tag{3}$$

It is clear that a discontinuity in α from +90° to −90° is not permitted since $\alpha = -90°$ violates condition (3). The arc beyond such a discontinuity is a nonminimal one. A continuous control, on the other hand, does satisfy condition (3). It may be noted that a discontinuity can be allowed only if the equality in Eq. (2) holds. The control given in Fig. 1 everywhere satisfies the Legendre-Clebsch condition.

In closing, it may be emphasized that the origin of the boundary value problem cannot be forgotten in seeking its solution. The Euler-Lagrange equations are not the only conditions that a minimizing arc must satisfy.

References

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† Only a portion of the α-optimal control is shown in Fig. 1.

Comment on "Wind-Tunnel Interference for Wing-Body Combination"

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I HAVE studied with interest Gorgui's analysis of the wind-tunnel interference for a wing-body combination.[1] Although I have obtained the same result for a circular tunnel, using the method of images, I believe that the results are misleading and the conclusion inaccurate.

It is usual to allow for the mean interference by means of a correction to the angle of attack, and one is then interested in the spanwise variation of interference which has not been accounted for by this correction.

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Now, the upwash induced by a change in angle of attack is not uniform over the span of a wing-body combination. Indeed, by considering uniform flow past a circular cylinder, it can be shown that the upwash is doubled in the vicinity of the wing-body junction. No allowance has been made in Gorgui's analysis for the change in body angle of attack, and this explains the variation in δ near the wing-body junction.

It appears that the curves for ($r/s = 0$) are valid for determining the correction to angle of attack and the residual interference for a wing-body combination. There is not, as suggested, any tendency towards a root stall, apart from that experienced at the corrected incidence in free flight. This conclusion is important, in view of the importance attached to stall development work in wind tunnels.

Reference

[1] Gorgui, M. A., "Wind-tunnel interference for wing-body combination," J. Aerospace Sci. 28, 823–825 (1961).

Correlation of the Critical Pressure of Conical Shells with That of Equivalent Cylindrical Shells

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IN Ref. 1, Seide showed that the critical pressures for isotropic conical shells under hydrostatic pressure can be correlated to those of equivalent cylindrical shells. The correlation yielded an approximate curve for the ratio of the critical pressure of conical shells to that of their equivalent cylindrical shells (Fig. 2 of Ref. 1). A very similar curve was obtained in Ref. 2 for conventional simple supports (which differ slightly from Seide's boundary conditions).

However, in both papers the calculations did not include large cone angles. Recent computations indicate that for larger cone angles the single curve should be replaced by a family of curves. Reappraisal of Fig. 2 of Ref. 1 brings out this cone angle dependence for 60°, as can be seen in Fig. 1, where that figure is reproduced with emphasis on the 60° points. (The remainder were for 10°, 20°, 30°, and 45°.) It is apparent that a better fitting correlation curve can be obtained if only cone angles up to 45° are included (or even better if only up to 30°) and the 60° points are joined by a similar curve.

Further computations for conventional simple supports by the method of Ref. 2 yields a family of correlation curves given in Fig. 2. The curves show the ratio of the critical pressure p of a conical shell to that of an equivalent cylindrical shell $\bar{p}$ vs the taper ratio ($1-R_1/R_2$). The equivalent cylindrical shell is defined as one having the same thickness as the conical shell, but whose radius is the mean radius of curvature of the cone and whose length is that of its slant length. As may be seen, the 60° curve deviates only slightly, whereas the 75° and 85° curves are noticeably lower. The actual percentage reduction in the ($p/\bar{p}$) ratio is only of the order of a few percent (up to about 6–7% for a large taper ratio and a cone angle of 85°), but since it is unconservative it is significant.

The computations brought out another nonconservative secondary effect. Seide's correlation curve[1] and that of Ref.

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